1. Harvard Law School courses often have assigned seating to facilitate the “Socratic method.”

Suppose that there are 100 first year Harvard Law students, and each takes two courses:

Torts and Contracts. Both are held in the same lecture hall (which has 100 seats), and the

seating is uniformly random and independent for the two courses.

(a) Find the probability that no one has the same seat for both courses (exactly; you should

leave your answer as a sum).

(b) Find a simple but accurate approximation to the probability that no one has the same

seat for both courses.

(c) Find a simple but accurate approximation to the probability that at least two students

have the same seat for both courses.

**(a) Probability that no one has the same seat for both courses:**

**To calculate this probability, we can employ the principle of inclusion-exclusion. We'll examine scenarios where students might occupy the same seats for both courses and then subtract that from 1 to determine the probability that no one shares a seat.**

**Let A\_i represent the event where student i occupies the same seat for both courses. The likelihood of any single student getting the same seat for both courses is 1/100 because there are 100 seats, each with equal probability.**

**Using the inclusion-exclusion principle:**

**P(at least one student shares the same seat for both courses) = 1 - P(no student shares the same seat for both courses)**

**P(no student shares the same seat for both courses) = 1 - [∑(P(A\_i) - ∑P(A\_i ∩ A\_j) + ∑P(A\_i ∩ A\_j ∩ A\_k) - ...)]**

**= 1 - [∑(1/100) - ∑(1/100 \* 1/99) + ∑(1/100 \* 1/99 \* 1/98) - ...]**

**This equation utilizes the inclusion-exclusion principle, and we must calculate it for all 100 students.**

**(b) Simple yet accurate approximation:**

**For a larger number of students (in this case, 100), the probability that no one shares the same seat for both courses can be approximated using the derangement formula (subfactorial):**

**D(n) = n!(1 - 1/1! + 1/2! - 1/3! + 1/4! - ... + (-1)^n/n!)**

**For n = 100, the approximation is:**

**D(100) ≈ 0.36787944**

**(c) Simple yet accurate approximation for at least two students sharing the same seat for both courses:**

**To find the probability that at least two students share the same seat, you can apply the complement rule:**

**P(at least two students share the same seat) = 1 - P(no student or exactly one student shares the same seat)**

**P(no student or exactly one student shares the same seat) ≈ D(100) + 100\*(1/100) = 1 + D(100)**

**Using the approximation from part (b):**

**P(at least two students share the same seat) ≈ 1 + 0.36787944 ≈ 1.3679**

2. There are 100 passengers lined up to board an airplane with 100 seats (with each seat

assigned to one of the passengers). The first passenger in line crazily decides to sit in a

randomly chosen seat (with all seats equally likely). Each subsequent passenger takes his or

her assigned seat if available, and otherwise sits in a random available seat. What is the

probability that the last passenger in line gets to sit in his or her assigned seat?

**The last passenger will get their assigned seat if and only if the first passenger sits in their assigned seat.**

**The probability that the first passenger selects the correct seat is 1/100 because there are 100 seats, and each seat is equally likely. Therefore, the probability that the last passenger gets their assigned seat is 1/100.**